Ceiling and Floor

\[
\lceil x \rceil = \text{Ceiling and means round up} \\
\lfloor x \rfloor = \text{Floor and means round down}
\]

Summation

\[
\sum_{i=0}^{n} f(i) = f(0) + f(1) + f(2) + f(3) + \ldots + f(n) \\
\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \\
\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \\
\sum_{i=0}^{n} a^i = \frac{a^{n+1} - 1}{a - 1}
\]

Definition of Big-O

\[ f(n) \in O(g(n)) \text{ if } \exists c \in R, c > 0 \text{ and } n_0 \in I, n_0 \geq 0, \text{ such that } \forall n \geq n_0 \quad f(n) \leq cg(n) \]

Useful Big-O Lemma

\[ f(n) \in O(g(n)) \text{ iff } \lim_{n \to \infty} \frac{f(n)}{g(n)} = c, c < \infty \]

Logarithms

\[ \log x = \log \text{ base 10 of } x \]
\[ \lg x = \log \text{ base 2 of } x \]
\[ \ln x = \log \text{ base e of } x \]

1. \( \log_a 1 = 0 \)
2. \( a^{\log_a x} = x \)
3. \( \log_a (xy) = \log_a x + \log_a y \)
4. \( \log_a (x/y) = \log_a x - \log_a y \)
5. \( \log_a x^y = y \cdot \log_a x \)
6. \( x^{\log_a y} = y^{\log_a x} \)
7. \( \log_a x = \log_{10} x / \log_{10} a \)

Master Theorem

\[ T(n) = \begin{cases} 
  c & \text{if } n < d \\
  aT(n/b) + f(n) & \text{if } n \geq d 
\end{cases} \]

Case 1: If there is a small constant \( c > 0 \), such that \( f(n) = O(n^{\log_a a - \varepsilon}) \), then \( T(n) = \Theta(n^{\log_a a}) \).

Case 2: If there is a constant \( k \geq 0 \), such that \( f(n) = \Theta(n^{\log_a a} \log^k n) \), then \( T(n) = \Theta(n^{\log_a a} \log^{k+1} n) \).

Case 3: If there are small constants \( \varepsilon > 0 \) and \( \delta < 1 \), such that \( f(n) = \Omega(n^{\log_a a + \delta \varepsilon}) \) and \( af(n/b) \leq \delta f(n) \), for \( n \geq d \), then \( T(n) = \Theta(f(n)) \).